Documentation of the Ground Equilibrium example for OpenGeoSys

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1 Problem formulation

Let there be a two-dimensional piece of ground (linear elastic soil saturated with water) with the following properties:

- soil and fluid density ρ_S and ρ_W,
- elastic soil properties E, ν,
- water viscosity μ,
- gravity field strength $\underline{g} = \text{-}g\underline{e}_y$,
- porosity ϕ.

At the left side and bottom the normal displacement and the normal hydraulic flux are zero (roller and no-flow BCs), whereas at the right side and the top the hydraulic pressure and the normal stress (traction) are prescribed according to the analytical solution (see Figure 1.

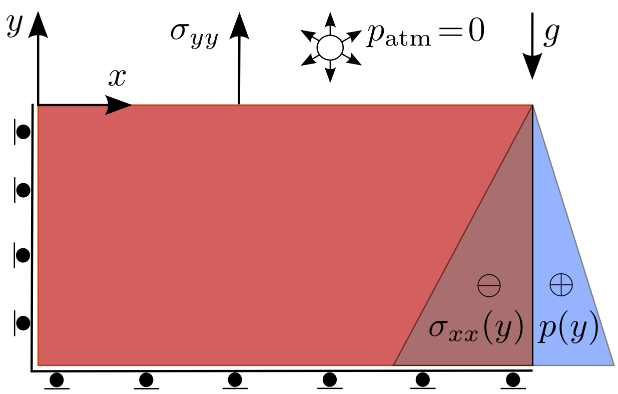

Figure 1: Two-dimensional ground model for the poroelastic simulation.

2 Analytical solution

Under these conditions there is the following homogeneous analytical solution.

Relations for the total stress:

There is a perfect lithostatic state of total stress $\underline{\underline{\sigma}} = \underline{\underline{\sigma}}' + \alpha(\text{-}p)\,\underline{\underline{I}}$ with non-zero coefficients

$$\sigma_{yy} = (\phi\rho_\text{W} + (1-\phi)\rho_\text{S})\, g\, y \;, \tag{1}$$

$$\sigma_{xx} = \sigma'_{xx} + \alpha(\text{-}p) = \sigma_{zz} \;. \tag{2}$$

Relations for the fluid pressure:

Considering the direction of the vertical coordinate y and the sign convention for pressure

$$p = \rho_\text{W} g(\text{-}y) \;. \tag{3}$$

Relations for the solid's effective stress:

The classical form of Hooke's law is only valid for the effective stress, i. e.

$$\underline{\underline{\sigma}}' = \underline{\underline{\underline{\underline{E}}}} : \underline{\underline{\varepsilon}} \;. \tag{4}$$

For the given BCs, the lithostatic state results in plane strain conditions where all strains are zero:

$$\varepsilon_{xx} = \varepsilon_{yy} = 0 \;. \tag{5}$$

Inserting this condition into Hooke's law gives

$$\varepsilon_{xx} = \frac{1-\nu^2}{E}\left[\sigma'_{xx} - \frac{\nu}{1-\nu}\,\sigma'_{yy}\right] = 0 \quad \leftrightarrow \quad \sigma'_{xx} = \frac{\nu}{1-\nu}\,\sigma'_{yy} \;. \tag{6}$$

Further, the effective vertical normal stress can be obtained combining Eq. (1) and (3) as

$$\sigma'_{yy} = \sigma_{yy} - \alpha(\text{-}p) = [(\phi-\alpha)\rho_\text{W} + (1-\phi)\rho_\text{S}]\, g\, y \;. \tag{7}$$

Finally, the total horizontal normal stress is obtained as

$$\begin{aligned} \sigma_{xx} &= \sigma'_{xx} + \alpha(\text{-}p) \\ &= \frac{\nu}{1-\nu}\,[(\phi-\alpha)\rho_\text{W} + (1-\phi)\rho_\text{S}]\, g\, y + \alpha\rho_\text{W} g\, y \\ &= \left\{\frac{\nu}{1-\nu}\,[(\phi-\alpha)\rho_\text{W} + (1-\phi)\rho_\text{S}] + \alpha\rho_\text{W}\right\} g\, y \;. \end{aligned} \tag{8}$$

3 Numerical example

In order to verify the numerical results, two tests were conducted with the same material parameters but different initial states ${}^0p, {}^0\underline{\underline{\sigma}}'$:

1. initial conditions according to the analytical solution $\rightarrow$ system is in equilibrium right from the start (static case),
2. zero initial conditions $\rightarrow$ the system evolves to analytical solution (transient case)

The material parameters listed in section 1 are chosen according to the following table:

Table 1: Material parameters for the numerical example

ρ_S / kg/m^3	E / Pa	ν	ρ_W / kg/m^3	μ / Pa s	ϕ	g / m/s^2	α
3000	$1 \cdot 10^6$	0.4	$1 \cdot 10^3$	$1 \cdot 10^{-3}$	0.2	9.81	1

4 Project file for *OpenGeoSys*

Note that in *OpenGeoSys* the traction BCs are given in terms of the *total* stress, whereas the initial stress state is given for the solid's *effective* stress:

$$\left[{}^0\sigma'\right] = \begin{bmatrix} \frac{\nu}{1-\nu} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{\nu}{1-\nu} \end{bmatrix} \left[(\phi - \alpha)\rho_\mathrm{W} + (1-\phi)\rho_\mathrm{S}\right] g\, y\,. \tag{9}$$

This can be defined in the project file in the following manner:

```
<processes>
  <process>
      <name>HM</name>
      <type>HYDRO_MECHANICS</type>
      ...
      <initial_stress>InitialEffectiveStressField</initial_stress>
  </process>
</processes>

...

<parameter>
  <name>InitialEffectiveStressField</name>
  <type>Function</type>
  <expression>0.4/(1-0.4)*((0.2-1.0)*1000+(1-0.2)*3000)*9.81*y</expression>
  <expression>            ((0.2-1.0)*1000+(1-0.2)*3000)*9.81*y</expression>
  <expression>0.4/(1-0.4)*((0.2-1.0)*1000+(1-0.2)*3000)*9.81*y</expression>
  <expression>0.0</expression><!--xy-->
</parameter>
```

The initial fluid pressure ${}^0p$ is given by (3) and can be defined straightforward as an initial condition of the (primary) pressure field. The boundary conditions at the top are $p = 0, \sigma_{yy} = 0$, whereas at the right boundary fluid pressure and *total* stress are given by Eq. (3) and (8), respectively.

The example can be extended easily to some ground indentation test replacing the zero-traction upper BC by some prescribed displacement curve. For the sake of testing the python functionality, a quadratic curve was chosen.