A benchmark of a viscoelastic(LUBBY2) model

The LUBBY2 model is based on the generalised Burgers model and is described by the following evolution equation (Nagel et al. (2016)):

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{C}_\mathrm{M} : \left[\dot{\boldsymbol{\epsilon}} - \boldsymbol{\mathcal{V}}_\mathrm{M}^{-1} : \boldsymbol{\sigma}^\mathrm{D} - \boldsymbol{\mathcal{V}}_\mathrm{K}^{-1} : (\boldsymbol{\sigma} - \boldsymbol{C}_\mathrm{K} : \boldsymbol{\epsilon}_\mathrm{K})\right] \tag{1}$$

$$\dot{\boldsymbol{\epsilon}}_\mathrm{K} = \boldsymbol{\mathcal{V}}_\mathrm{K}^{-1} : (\boldsymbol{\sigma} - \boldsymbol{C}_\mathrm{K} : \boldsymbol{\epsilon}_\mathrm{K}) \tag{2}$$

$$\dot{\boldsymbol{\epsilon}}_\mathrm{M} = \boldsymbol{\mathcal{V}}_\mathrm{M}^{-1} : \boldsymbol{\sigma} \tag{3}$$

where $\boldsymbol{\mathcal{V}}_\mathrm{M}$ and $\boldsymbol{\mathcal{V}}_\mathrm{K}$ represent the viscosity tensor of the Maxwell and Kelvin model, respectively. $\boldsymbol{C}_\mathrm{M}$ and $\boldsymbol{C}_\mathrm{K}$ are the tangent moduli. The Kelvin shear modulus and the viscosities are functions of the current stress state:

$$G_\mathrm{K} = G_\mathrm{K0} e^{m_\mathrm{K} \sigma_\mathrm{eff}} \tag{4}$$

$$\eta_\mathrm{M} = \eta_\mathrm{M0} e^{m_1 \sigma_\mathrm{eff}} \tag{5}$$

$$\eta_\mathrm{K} = \eta_\mathrm{K0} e^{m_2 \sigma_\mathrm{eff}} \tag{6}$$

with

$$\sigma_\mathrm{eff} = \sqrt{\frac{3}{2} \boldsymbol{\sigma}^\mathrm{D} : \boldsymbol{\sigma}^\mathrm{D}} \tag{7}$$

where m_a are material parameters characterising the stress dependency.
The rheological model is shown in Fig. 1 consisting of a Maxwell element in series with a Kelvin element.

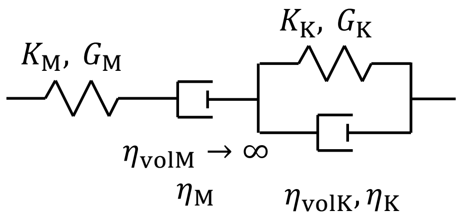


Figure 1: Rheological analogue of the LUBBY2 model.

The residual vectors read with the state vector $z = \left(\boldsymbol{\sigma}^\mathrm{DT}, \boldsymbol{\epsilon}_\mathrm{M}^\mathrm{DT}, \boldsymbol{\epsilon}_\mathrm{K}^\mathrm{DT}, \eta_\mathrm{M}^\mathrm{T}, \eta_\mathrm{K}^\mathrm{T}, G_\mathrm{K}^\mathrm{T}\right)^\mathrm{T}$:

$$\boldsymbol{r}_1^j = \boldsymbol{\sigma}^{\mathrm{D}j} - 2\left(\boldsymbol{\epsilon}^{\mathrm{D}j} - \boldsymbol{\epsilon}_\mathrm{K}^{\mathrm{D}j} - \boldsymbol{\epsilon}_\mathrm{M}^{\mathrm{D}j}\right) \tag{8}$$

$$\boldsymbol{r}_2^j = \frac{\boldsymbol{\epsilon}_\mathrm{K}^{\mathrm{D}j} - \boldsymbol{\epsilon}_\mathrm{K}^{\mathrm{D}t}}{\Delta t} - \frac{1}{2\eta_\mathrm{K}}\left(G_\mathrm{M}\boldsymbol{\sigma}^{\mathrm{D}j} - 2G_\mathrm{K}\boldsymbol{\epsilon}_\mathrm{K}^{\mathrm{D}j}\right) \tag{9}$$

$$\boldsymbol{r}_3^j = \frac{\boldsymbol{\epsilon}_\mathrm{M}^{\mathrm{D}j} - \boldsymbol{\epsilon}_\mathrm{M}^{\mathrm{D}t}}{\Delta t} - \frac{\eta_M}{2G_\mathrm{M}}\boldsymbol{\sigma}^{\mathrm{D}j} \tag{10}$$

and the 18×18 Jacobian:

$$\frac{\partial \boldsymbol{G}}{\partial z} = \begin{pmatrix} \boldsymbol{J}_{11} & \boldsymbol{J}_{12} & \boldsymbol{J}_{13} \\ \boldsymbol{J}_{21} & \boldsymbol{J}_{22} & \boldsymbol{J}_{23} \\ \boldsymbol{J}_{31} & \boldsymbol{J}_{32} & \boldsymbol{J}_{33} \end{pmatrix} \tag{11}$$

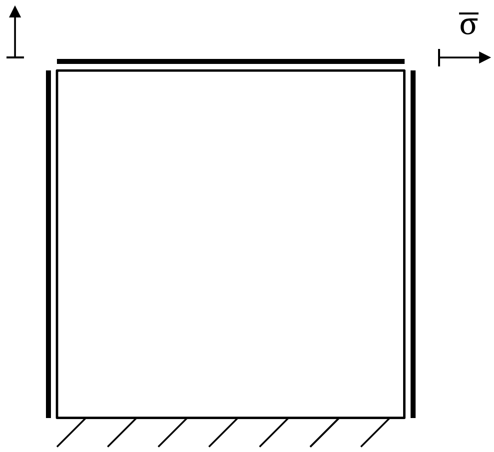


Figure 2: Loading and boundary conditions.

where the components are given as follows:

$$\boldsymbol{J}_{11} = \boldsymbol{I}, \quad \boldsymbol{J}_{12} = 2\boldsymbol{I}, \quad \boldsymbol{J}_{13} = 2\boldsymbol{I} \tag{12}$$

$$\boldsymbol{J}_{21} = -\frac{G_\mathrm{M}}{2\eta_K}\boldsymbol{I}, \quad \boldsymbol{J}_{22} = \frac{1}{\Delta t}\boldsymbol{I}, \quad \boldsymbol{J}_{23} = \boldsymbol{0} \tag{13}$$

$$\boldsymbol{J}_{31} = -\frac{G_\mathrm{M}}{2\eta_M}\boldsymbol{I}, \quad \boldsymbol{J}_{32} = \boldsymbol{0}, \quad \boldsymbol{J}_{33} = \frac{1}{\Delta t}\boldsymbol{I} \tag{14}$$

$$\tag{15}$$

for $\sigma_\mathrm{eff} > 0$

$$\boldsymbol{J}_{21} = \frac{1}{2\eta_\mathrm{K}}\left(G_\mathrm{M}\boldsymbol{\sigma}^{\mathrm{D}j} - 2G_K\boldsymbol{\epsilon}^{\boldsymbol{j}}_{\mathbf{K}}\right)\frac{3}{2}m_2 G_\mathrm{M}\frac{\left(\boldsymbol{\sigma}^{\mathrm{D}j}\right)^\mathrm{T}}{\sigma_\mathrm{eff}} + \frac{3}{2\eta_\mathrm{K}}\boldsymbol{\epsilon}^j_\mathrm{K} m_\mathrm{K} G_\mathrm{K} G_\mathrm{M}\frac{\left(\boldsymbol{\sigma}^{\mathrm{D}j}\right)^\mathrm{T}}{\sigma_\mathrm{eff}} \tag{16}$$

$$\boldsymbol{J}_{31} = \frac{1}{2\eta_\mathrm{M}}G_\mathrm{M}\boldsymbol{\sigma}^{\mathrm{D}j}\frac{3}{2}m_1 G_\mathrm{M}\frac{\left(\boldsymbol{\sigma}^{\mathrm{D}j}\right)^\mathrm{T}}{\sigma_\mathrm{eff}} \tag{17}$$

$$\tag{18}$$

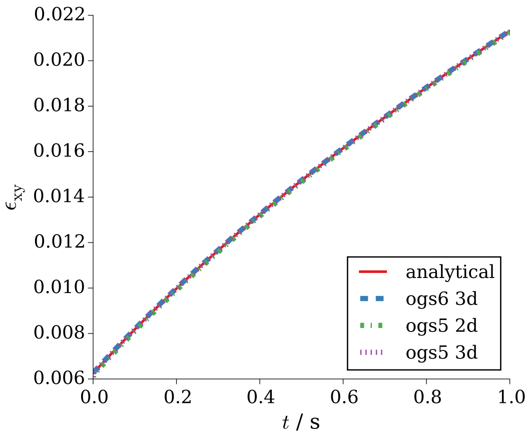
The mechanical model is a square plate/cube with a positive shear stress of 0.01 MPa applied on the top side/surface, see Fig. 2. Displacements of the left, right side and the top are constrained in vertical direction. The material property set for this benchmark is listed in Table 1.

Table 1: Material properties used in LUBBY2 model

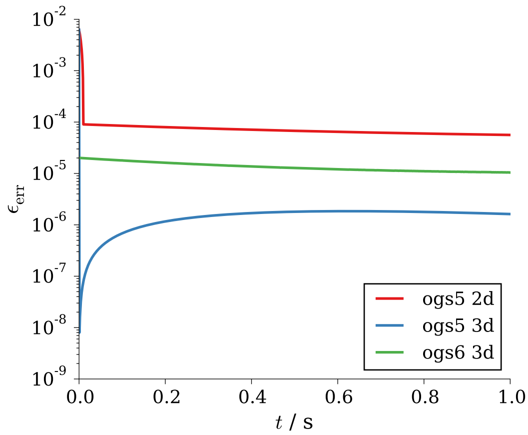
G_M / MPa	K_M / MPa	η_M0 / MPa d	G_K0 / MPa	η_K0 / MPa d	m_1 / MPa^{-1}	m_2 / MPa^{-1}	m_G / MPa^{-1}
0.8	0.8	0.5	0.8	0.5	-0.3	-0.2	-0.2

References

Nagel, T., Minkley, W., Bö, N., Görke, U.-J., Kolditz, O., 2016. Implicit numerical integration and consistent linearisation of inelastic constitutive models of salt rock behaviour. ...... in press.



(a) ϵ_{xy}.



(b) deviation.

Figure 3: Variation of the shear strain with time (a) and the deviation between analytical solution and numerical simulations (b).