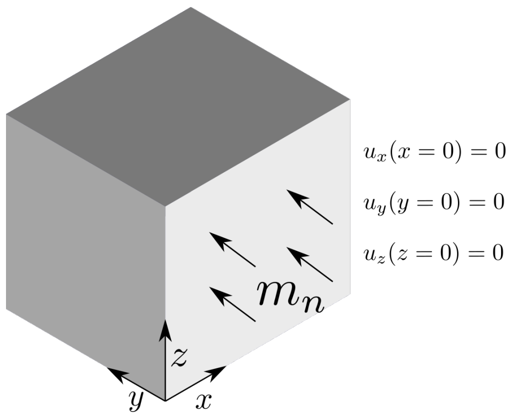
Hydromechanics: flow, free expansion



This test uses a constant fluid influx with free expansion of the solid.

$$0 = (\varrho_{\mathrm{F}})'_{\mathrm{S}} + \mathrm{div}(\varrho_{\mathrm{FR}}\underline{\tilde{w}}_{\mathrm{FS}}) + \varrho_{\mathrm{F}}\,\mathrm{div}(\underline{u}) \tag{13}$$

Assuming a linear elastic behaviour of the solid and the applicability of the ideal gas law the following relation can be obtained

$$(\varrho_{\mathrm{F}})'_{\mathrm{S}} = \varrho_{\mathrm{FR}}\beta_p\,(p_{\mathrm{FR}})'_{\mathrm{S}}\,\phi + \varrho_{\mathrm{FR}}(\alpha_{\mathrm{B}}-\phi)\left[\mathrm{div}\,(\underline{u})'_{\mathrm{S}} + K_{\mathrm{SR}}^{-1}\,(p_{\mathrm{FR}})'_{\mathrm{S}}\right] \tag{14}$$
$$(\varrho_{\mathrm{F}})'_{\mathrm{S}} = \varrho_{\mathrm{FR}}\left[(p_{\mathrm{FR}})'_{\mathrm{S}}\,(\beta_p\phi + K_{\mathrm{SR}}^{-1}(\alpha_{\mathrm{B}}-\phi)) + \mathrm{div}\,(\underline{u})'_{\mathrm{S}}\,(\alpha_{\mathrm{B}}-\phi)\right] \tag{15}$$

With $\alpha_{\mathrm{B}}\,(p_{\mathrm{FR}})'_{\mathrm{S}} = K_{\mathrm{S}}\,(e)'_{\mathrm{S}} = K_{\mathrm{S}}\,\mathrm{div}\,(\underline{u})'_{\mathrm{S}}$ we get

$$(\varrho_{\mathrm{F}})'_{\mathrm{S}} = \varrho_{\mathrm{FR}}\left[(p_{\mathrm{FR}})'_{\mathrm{S}}\,(\beta_p\phi + K_{\mathrm{SR}}^{-1}(\alpha_{\mathrm{B}}-\phi)) + \alpha_{\mathrm{B}}K_{\mathrm{S}}^{-1}\,(p_{\mathrm{FR}})'_{\mathrm{S}}\,(\alpha_{\mathrm{B}}-\phi)\right] \tag{16}$$
$$(\varrho_{\mathrm{F}})'_{\mathrm{S}} = \varrho_{\mathrm{FR}}\,(p_{\mathrm{FR}})'_{\mathrm{S}}\left[\beta_p\phi + (K_{\mathrm{SR}}^{-1} + \alpha_{\mathrm{B}}K_{\mathrm{S}}^{-1})(\alpha_{\mathrm{B}}-\phi)\right] \tag{17}$$
$$(\varrho_{\mathrm{F}})'_{\mathrm{S}} = \varrho_{\mathrm{FR}}\,(p_{\mathrm{FR}})'_{\mathrm{S}}\left[\beta_p\phi + K_{\mathrm{S}}^{-1}(\alpha_{\mathrm{B}}-\phi)\right] \tag{18}$$
$$(\varrho_{\mathrm{F}})'_{\mathrm{S}} = (p_{\mathrm{FR}})'_{\mathrm{S}}\,(R_sT)^{-1}\left[\phi + p_{\mathrm{FR}}K_{\mathrm{S}}^{-1}(\alpha_{\mathrm{B}}-\phi)\right] \tag{19}$$

$$-\int\limits_{\Omega}\mathrm{div}(\varrho_{\mathrm{FR}}\underline{\tilde{w}}_{\mathrm{FS}})\,\mathrm{d}\Omega = \int\limits_{\Omega}(\varrho_{\mathrm{F}})'_{\mathrm{S}} + \varrho_{\mathrm{F}}\,\mathrm{div}\,(u)'_{\mathrm{S}}\,\mathrm{d}\Omega \tag{20}$$
$$R_sTm_n\frac{\Gamma}{\Omega} = (p_{\mathrm{FR}})'_{\mathrm{S}}\left[\phi + p_{\mathrm{FR}}K_{\mathrm{S}}^{-1}(\alpha_{\mathrm{B}}-\phi)\right] + \varrho_{\mathrm{FR}}\phi\alpha_{\mathrm{B}}K_{\mathrm{S}}^{-1}\,(p_{\mathrm{FR}})'_{\mathrm{S}} \tag{21}$$
$$R_sTm_n\frac{\Gamma}{\Omega} = (p_{\mathrm{FR}})'_{\mathrm{S}}\left[\phi + p_{\mathrm{FR}}K_{\mathrm{S}}^{-1}(\alpha_{\mathrm{B}}-\phi) + p_{\mathrm{FR}}\phi\alpha_{\mathrm{B}}K_{\mathrm{S}}^{-1}\right] \tag{22}$$
$$R_sTm_n\frac{\Gamma}{\Omega} = (p_{\mathrm{FR}})'_{\mathrm{S}}\left[\phi + p_{\mathrm{FR}}K_{\mathrm{S}}^{-1}\,(\alpha_{\mathrm{B}}-\phi+\alpha_{\mathrm{B}}\phi)\right] \tag{23}$$
$$R_sTm_n\frac{\Gamma}{\Omega} = (p_{\mathrm{FR}})'_{\mathrm{S}}\left[\phi + p_{\mathrm{FR}}K_{\mathrm{S}}^{-1}\psi\right] \tag{24}$$
$$\text{With}\qquad \psi = \alpha_{\mathrm{B}} - \phi + \alpha_{\mathrm{B}}\phi \tag{25}$$

$$\int_{t_0}^{t} R_s T m_n \frac{\Gamma}{\Omega}\,\mathrm{d}t = \int_{p_{\mathrm{FR0}}}^{p_{\mathrm{FR}}} \phi + p_{\mathrm{FR}} K_{\mathrm{S}}^{-1}\psi\,\mathrm{d}p_{\mathrm{FR}} \tag{26}$$

$$R_s T m_n \frac{\Gamma}{\Omega} t = \phi(p_{\mathrm{FR}} - p_{\mathrm{FR0}}) + \frac{\psi}{2K_{\mathrm{S}}}\left(p_{\mathrm{FR}}^2 - p_{\mathrm{FR0}}^2\right) \tag{27}$$

$$0 = p_{\mathrm{FR}}^2 + 2K_{\mathrm{S}}\frac{\phi}{\psi}p_{\mathrm{FR}} - p_{\mathrm{FR0}}^2 - 2K_{\mathrm{S}}\frac{\phi}{\psi}p_{\mathrm{FR0}} - 2K_{\mathrm{S}}\frac{1}{\psi}R_s T m_n \frac{\Gamma}{\Omega} t \tag{28}$$

$$p_{\mathrm{FR}} = -K_{\mathrm{S}}\frac{\phi}{\psi} \pm \sqrt{\left(K_{\mathrm{S}}\frac{\phi}{\psi} + p_{\mathrm{FR0}}\right)^2 + 2K_{\mathrm{S}}\frac{1}{\psi}R_s T m_n \frac{\Gamma}{\Omega} t} \tag{29}$$

The strain solution can be obtained by integrating the equation

$$K_{\mathrm{S}}\,(e)'_{\mathrm{S}} = \alpha_{\mathrm{B}}\,(p_{\mathrm{FR}})'_{\mathrm{S}} \tag{30}$$

$$\int_{e_0}^{e} \mathrm{d}e = \frac{\alpha_{\mathrm{B}}}{K_{\mathrm{S}}}\int_{p_{\mathrm{FR0}}}^{p_{\mathrm{FR}}} \mathrm{d}p_{\mathrm{FR}} \tag{31}$$

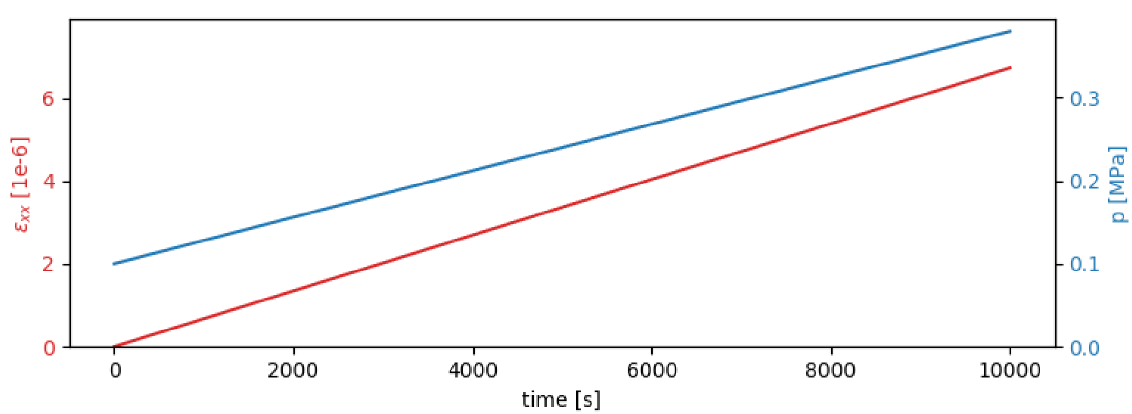
$$e = \frac{\alpha_{\mathrm{B}}}{K_{\mathrm{S}}}(p_{\mathrm{FR}} - p_{\mathrm{FR0}}) \tag{32}$$

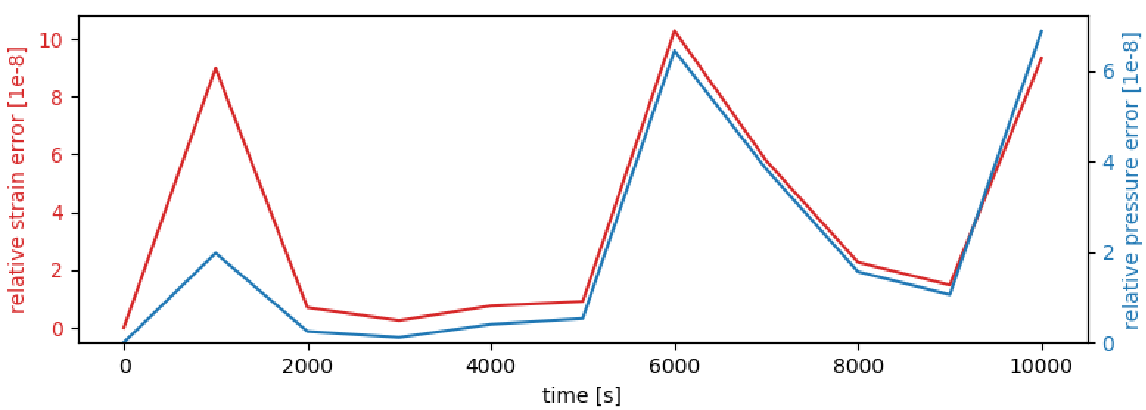
$$e = 3\varepsilon_{xx} = 3\varepsilon_{yy} = 3\varepsilon_{zz} \tag{33}$$

Given the following parameters

$$\begin{aligned}
\Gamma &= 1\,\mathrm{m}^2 & \Omega &= 1\,\mathrm{m}^3 \\
R &= 287.058\,\mathrm{J/(kgK)} & T &= 293.15\,\mathrm{K} \\
E &= 10\,000\,\mathrm{MPa} & \nu &= 0.3 \\
p_{\mathrm{FR0}} &= 0.1\,\mathrm{MPa} & m_n &= 10^{-4}\,\mathrm{kg/(m^2 s)} \\
\phi &= 0.3 & \alpha_{\mathrm{B}} &= 0.6 \\
K_{\mathrm{SR}} &= \frac{K_{\mathrm{S}}}{1-\alpha_{\mathrm{B}}} & K_{\mathrm{S}} &= \frac{E}{3(1-2\nu)}
\end{aligned}$$

we get the following pressure and strain solution for a time of 10 000 s. The simulation result is reasonable close to the analytical solution with the maximum relative error being about 10^{-7} in both instances.




relative strain error [1e-8]
relative pressure error [1e-8]
time [s]
0
2
4
6
8
10
0
2000
4000
6000
8000
10000